Enrollment No: ______________________ **Exam Seat No:** ______________________

C. U. SHAH UNIVERSITY

Winter Examination - 2022

Subject Name : Metric Space

Subject Code: 4SC05MES1 **Branch: B.Sc. (Mathematics)**

Semester: 5 **Date: 24/11/2022** **Time: 02:30 To 05:30** **Marks: 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **[14]**

a) Let (X, d) be a metric space and $E \subset X$.Then set E is said to be closed set if … **(01)**
1) $E^{'} = X$
2) $E^{'} = \emptyset$
3) $\bar{E} = X$
4) $\bar{E} = E$

b) Which of the following subset of $\boldsymbol{R}$is neither open nor closed? **(01)**
1) (0,1]
2) $\boldsymbol{R}$
3) $\emptyset$
4) None of these

c) Let (X, d) be a discrete metric space and $r > 1$,then $S_r(x) =$ _____ **(01)**
1) $\{x\}$
2) X
3) (0,1)
4) [0,1]

d) Define : Connected Set. **(01)**

e) Define : Limit Point. **(01)**

f) Check whether the statement is true or false: $\bar{A} \cap \bar{B} \subseteq \overline{A \cap B}$. **(01)**

g) Define : Metric Space. **(01)**

h) Check whether the statement is true or false: $(1,2)$ and $(-1,3)$ are separated subsets of metric space R. **(01)**

i) Find $\bar{A}$ for $A = [-1,3)$. **(01)**

j) Check whether the statement is true or false: Let A be connected subset of metric space X and B be a subset of X such that $A \subseteq B \subseteq \bar{A}$ then B is also connected. **(01)**

k) Let $X = \boldsymbol{R}$and $A = \emptyset$ then find $int\ A$ and $ext\ A$. **(02)**

l) Define : Continuous function in Metric space . **(02)**

Attempt any four questions from Q-2 to Q-8.

Q-2 Attempt all questions [14]

a) Let(X, d)be a metric space and $d_1 : X \times X \to \boldsymbol{R}$defined by $d_1(x, y) = \frac{d(x,y)}{1+d(x,y)}$then prove that $d_1$is also a metric on X. **(06)**

b) Prove : Arbitrary union of open sets of metric space is an open set. **(05)**

c) Show that every finite subset of metric space is closed. **(03)**

Q-3 Attempt all questions [14]

a) State and prove Cantor's Intersection Theorem. **(06)**

b) Let d be matrix on non-empty set X. Define $d^* : R \times R \to R$ by $d^*(x, y) = 2\, d(x, y)$ then prove that (X, d^*) is also metric space. **(05)**

c) Let $X = [0,1]$ with usual metric space. Find $S_1\left(\frac{1}{2}\right), S_{\frac{1}{3}}(0)$ and $S_{\frac{3}{32}}\left(\frac{1}{32}\right)$. **(03)**

Q-4 Attempt all questions [14]

a) Prove that the image of Cauchy sequence under uniformly continuous function is again a Cauchy sequence. Can we replace'uniform continuous' by 'continuous' in this statement? **(06)**

b) Prove that in a metric space every open sphere is an open set. **(05)**

c) If (X, d) is a metric space and $A, B \subset X$ then show that$(A \cup B)' \subseteq A' \cup B'$. **(03)**

Q-5 Attempt all questions [14]

a) For a non-empty subset A of metric space (X, d) show that the function $f : X \to \boldsymbol{R}$defined by $f(x) = d(x, A)$, $x \in X$ is uniformly continuous. Also show that $f(x) = 0$ if and only if $x \in \bar{A}$. **(07)**

b) Prove that any contraction mapping f of non-empty complete metric space (X, d) into itself has a unique fixed point. **(07)**

Q-6 Attempt all questions [14]

a) Let (X, d_1) and (Y, d_2) be any two metric spaces, then prove that $f : X \to Y$ is continuous if and only if $f^{-1}(F)$ is closed in X whenever F is closed in Y. **(07)**

b) Prove : Continuous image of compact set is compact set. **(07)**

Q-7 Attempt all questions [14]

a) Let Y be a subset of metric space (X, d) then show that following are equivalent : **(10)**

(i) Y is connected.

(ii) Y can't express as disjoint union of two non-empty closed sets.

(iii) $\emptyset$ and Y are only sets which are both open and closed in Y.

b) Let (X, d) be a discrete metric space then prove that every function $f : X \to Y$ is continuous on X . **(04)**

Q-8 Attempt all questions [14]

a) Show that every closed and bounded subset of real line is compact. **(09)**

b) Define Cantor set and show that cantor is a closed set. **(05)**